Geometry, Fall 2012

University of Maryland, Department of Mathematics course 430

HW 3: (due by September 27, in class (except for problem 7—see below).)

Please write-up your own solutions to problems in an organized and neat fashion. If collaborating in the problem solving process please write the names of the people with whom you collaborated next to each solution.

0. Recall the following approximate observations discussed in class:
(i) during full moon, the moon takes about 2 minutes to completely set from the moment it first touches the horizon;
(ii) The distance $d_{M,E}$ from the moon to the Earth is approximately $60R_E$ where R_E is the radius of the Earth.
Let R_M denote the radius of the moon. Give an approximation for R_M in terms of R_E.

1. Complete the proof given in class of the Three Reflections Theorem. Namely, show that
(i) every isometry of $\mathbb{R}^2$ that fixes exactly one point is a composition of two reflections (and describe precisely those reflections),
(ii) every isometry of $\mathbb{R}^2$ that fixes no point is a composition of three reflections (and again describe precisely those reflections).

2. (Stillwell, Geometry of Surfaces, problem 1.4.1) Show that any isometry of $\mathbb{R}^3$ is the composition of up to four reflections (here we mean reflections about a plane in $\mathbb{R}^3$).

3. (Stillwell, Geometry of Surfaces (GOS), problem 1.5.3) Deduce from the classification of isometries that each isometry of $\mathbb{R}^2$ has either: (i) a line of fixed points, (ii) a single fixed point, (iii) no fixed points and a parallel family of invariant lines (an invariant set is a set mapped onto itself under the map), or (iv) no fixed points, and a single invariant line.

4. (Stillwell, GOS, problems 3.1.1) A great circle in the unit sphere $S^2 \subset \mathbb{R}^3$ is defined as the intersection of a plane through the orign with S^2.
Show that two great circles on $S^2 \subset \mathbb{R}^3$ intersect at two antipodal points, i.e., at (x, y, z) and $(-x, -y, -z)$.

5. (Stillwell, GOS, problems 3.2.1) Let $r_1, r_2 \in \text{Isom}(S^2)$ be rotations. Under what conditions is $r_1r_2 = r_2r_1$?

6. (Stillwell, GOS, problems 3.2.2) Find a product of three reflections with no fixed point.

7. This one is a lot of fun. Construct (by hand) real (3D) models of the five regular polyhedra. This problem will be graded as follows: the models should be constructed by each student (collaboration is allowed, but each student should construct his own models), and brought to class on Tuesday October 9 (the day of the first midterm). Each successful model will count towards 2 points (out of 25 total for the midterm). **Bringing <u>all five</u> models will be a prerequisite for taking the midterm that day.**