Differential Geometry I, Autumn 2013

University of Maryland, Department of Mathematics course 436

HW 1: (due by September 24, in class.)

Please write-up your own solutions to problems in an organized and neat fashion. Collaborating in the problem solving process is not allowed for this homework.

0. Callahan: Exercises 5,7,8,10 on pages 218–221.

1.

Definition. The *round 2-sphere* is the subset of $\mathbb{R}^3$ parametrized by $S^2 := \{(x_1, x_2, x_3) : x_1^2 + x_2^2 + x_3^2 = 1\}$ endowed with the metric induced from $(\mathbb{R}^3, dx_1^2 + dx_2^2 + dx_3^2)$. We denote this induced metric by g.

(a) Exhibit explicitly coordinate charts for S^2. (You can use, e.g., stereographic projection.)

(b) Write down explicitly the transition maps between these charts as well as their inverses.

(c) Write down the differential of the transition maps and their inverses.

(d) consider the curve γ concatenated from the following three segments: the quarter great circle from the North pole to equator; a half circle along the equator; and a quarter circle down to the South pole. Compute the length of γ *by working in the charts* from (a) (and *not* by simply computing the length of this using much simpler considerations). In particular, show how the computations can be carried out in either or both charts to give the same answer.

2. Let M be a differentiable manifold and let TM denote its tangent bundle (as defined in class). Let $\pi : TM \to M$ denote the natural projection map and let $p \in M$. Show that $\pi^{-1}(p) = \mathbb{R}^n$.