Geometric Analysis, Autumn 2014

University of Maryland, Department of Mathematics course 742

HW 2: (due by October 23.)

1.
Show that the functional $\int |\nabla u|^2 + \lambda u^2$ is weakly sequentially l.s.c. on $W_0^{1,2}(\Omega)$, with notation as in class.

2.
Complete the proof, initiated in class, of the fact that the constrained minimizer of the functional $\int |\nabla u|^2 + \lambda u^2$ (satisfying $\int u^p = 1$) can be appropriated rescaled to give a solution of $-\Delta u + \lambda u = |u|^{p-2}u$.

3.
Let g be a Riemannian metric on M. Obtain a PDE in terms of the positive smooth function u relating the scalar curvature of $u^{\frac{4}{n-2}}g$ to that of g.