The Yamabe Problem, Spring 2013

University of Maryland, Department of Mathematics course 748F

HW4:

Please do not submit this.

1. Suppose that g, h are two metrics in the same conformal class satisfying $R_g \leq R_h < 0$. Prove that $g/h \leq 1$. Is this true also in a conformal class with a positive Yamabe invariant? How about zero?

2. Let $S^k(t)$ denote the standard round metric induced on the radius t sphere in $\mathbb{R}^{k+1}$. Compute the scalar curvature of the Riemannian product space $S^m(r) \times S^n(R)$. (Good reference: Petersen, Riemannian Geometry.)

3. Suppose $M = S^1(t) \times S^{n-1}(1)$ and denote by g_0 the associated product metric on M.
(i) Assume that u is a function of the first factor alone. Write down the ODE satisfied by u if $g = u^{\frac{4}{n-2}} g_0$ has constant scalar curvature (By the way, it is possible to show, using a certain maximum principle type argument, that the assumption is always satisfied when g has csc.)
(ii) Find the associated Hamiltonian.
(iii) What is the value of u such that the associated metric is a product of round spheres, and has scalar curvature equal to $n(n-1)$?
(iv) Prove that $[g_0]$ does not admit any metrics conformal to $S^1(s) \times S^{n-1}(1)$ with $s \neq t$.