Kähler manifolds, Autumn 2014

University of Maryland, Department of Mathematics course 868K

HW 4: (due by October 23.)

1. (Invariance of $h^{p,0}$)
Let M and N be birationally isomorphic. Prove that $h^{0,p}(N) = h^{0,p}(M)$. (This was only sketched in class; in particular, *prove* in detail the version of Hartog's theorem that you will be using.)

2.
Prove that indeed all quadric hypersurfaces in $\mathbb{P}^n$ are biholomorphic.

3.
(a) Complete the proof, initiated in class, that the quadric surface in $\mathbb{P}^3$ is biholomorphic to $\mathbb{P}^1 \times \mathbb{P}^1$ (This proof should use Problem 2 and computation in coordinate charts.)

(b) Give full details to the concluding remark in the proof given in class that S is biholomorphic to $\mathbb{P}^1 \times \mathbb{P}^1$ (given the facts established in class concerning A-lines and B-lines).

4.
Give full details to the claim on p. 491 of Griffiths–Harris, that if M is complex, and $V \subset M$ is an analytic subvariety of codimension at least 2, then any holomorphic map $f : M \backslash V \to \mathbb{P}^n$ admits a sort of meromorphic extension to all of M.

5.
Let S be a quadric surface in $\mathbb{P}^3$, and let $p \in S$. Let π_p be the unique holomorphic extension of the (rational) projection map from $\pi_p : S \setminus \{p\}$ to a $\mathbb{P}^2$ sitting in $\mathbb{P}^3$. Show that π_p maps the exceptional divisor of the blow-up E to the line passing through the points $\pi_p(L_1), \pi_p(L_2) \in \mathbb{P}^2$, where L_i are the unique lines contained in S and passing through p.