HW 5:

1. Prove or disprove the following claims related to the proof of the Kähler identities:
   A) $t_k e_l + e_l t_k = 0$.
   B) $\bar{t}_k e_l + e_l \bar{t}_k = 0$.

2. Show that $[\Lambda, L] \circ \Pi^{p,q} = (n - p - q)\Pi^{p,q}$.

3. Let $M$ be a projective surface and let $C$ be a -1-curve in $M$. Denote by $\pi$ the map furnished by the Castelnuovo–Enriques Theorem. Give a detailed proof that:
   A) $\pi(C)$ is a point.
   B) $\pi(M)$ is smooth at $\pi(C)$.

4. Follow through the steps of the proof of the Castelnuovo–Enriques theorem for $M$ being the blowup of $\mathbb{P}^2$ at one point. Namely, use the explicit description of the blowup to embed this manifold in a projective space, identify the -1-curve, and then construct the map $\pi$ and prove that its image is $\mathbb{P}^2$. 