Math 141 Midterm 1 Question 1 Solution

September 23, 2015

**Question:** Find the volume of the solid obtained by revolving about the $y$ axis the region between the graphs of functions $f(x) = \sqrt{3 + x}$ and $g(x) = \frac{\ln(x)}{x^2}$ on the interval $[1, 3]$.

**Solution:**
Since we revolve around the $y$ axis and have our equations in terms of $x$ we will use the shell method. We see that on $[1, 3]$

$$f(x) > g(x)$$ 3 pts

thus

$$V = \int_{1}^{3} 2\pi x \left( \sqrt{3 + x} - \frac{\ln(x)}{x^2} \right) dx$$ 7 pts

Pull the $2\pi$ out and integrate as two separate integrals. For $\int_{1}^{3} x\sqrt{x + 3} \, dx$ set $u = 3 + x$ so $du = dx$ and $x = u - 3$. For $\int_{1}^{3} \frac{\ln(x)}{x} \, dx$ set $w = \ln(x)$ so $dw = \frac{1}{x} \, dx$. Plug the substitutions into the bounds and we have

$$2\pi \left[ \int_{4}^{6} u \sqrt{u} \, du - \int_{\ln(3)}^{0} w \, dw \right].$$ 5 pts

Integrating now gives

$$2\pi \left[ \frac{2}{5} u^{\frac{5}{2}} - 2u^2 \big|_{4}^{6} - \frac{1}{2} w^2 \big|_{\ln(3)}^{0} \right].$$ 7 pts

Finally plugging in our bounds gives

$$V = 2\pi \left[ \frac{2}{5} 6^\frac{5}{2} - 2 \cdot 6^2 - \frac{2}{5} 4^\frac{5}{2} + 2 \cdot 4^2 - \frac{1}{2} (\ln(3))^2 \right]$$ 3 pts

and we’ll leave our answer as that.