Math 141 Midterm 1 Question 2 Solution

September 24, 2014

**Question:** Find the largest set which contains 0 and on which the function \( f(x) = x^9 + 6x^3 + 2x - 10 \) has an inverse. Find the derivative of \( f^{-1} \) (the inverse of \( f \)) at \( y = -10 \).

**Solution:**

Take the derivative first

\[
f'(x) = 9x^8 + 18x^2 + 2 \quad 4 \text{ pts}
\]

See that

\[
f'(x) > 0 \quad 5 \text{ pts}
\]

and thus \( f(x) \) is always increasing.

Thus the largest set which contains 0 is

\((-\infty, \infty)\). \quad 5 \text{ pts}

To find \( (f^{-1})'(-10) \) use the formula

\[
(f^{-1})'(c) = \frac{1}{f'(a)}.
\]

where \( f(a) = c \).

By inspection find \( a \) such that \( f(a) = -10 \)

\[-10 = a^9 + 6a^3 + 2a - 10 \quad 4 \text{ pts}\]

so \( a = 0 \).

Therefore we have

\[
(f^{-1})'(-10) = \frac{1}{f'(0)} = \frac{1}{2}. \quad 2 \text{ pts}
\]