Problem 2: \( \sum_{n \in \mathbb{N}} \frac{2^n}{n3^n} x^{2n} \)

\[ a_n = \frac{2^n}{n3^n} |x|^{2n}, \quad a_{n+1} = \frac{2^{n+1}}{3^{n+1}(n+1)} |x|^{2n+2} \quad [1] \]

Applying the ratio test:

\[ \frac{a_{n+1}}{a_n} = \frac{2}{3} |x|^2 \frac{n}{n+1} \quad [2] \]

\[ \lim_{n \to \infty} \frac{a_{n+1}}{a_n} = \frac{2}{3} |x|^2 \quad [3] \]

Or, applying the root test:

\[ \sqrt[n]{a_n} = \frac{2 |x|^2}{3 \sqrt{n}} \quad [2] \]

\[ \lim_{n \to \infty} \sqrt[n]{a_n} = \frac{2}{3} |x|^2 \quad [3] \]

This limit must be \(<1 \Rightarrow |x| < \frac{\sqrt{3}}{2} \)

\( \Rightarrow \) the radius of convergence is \( \frac{\sqrt{3}}{2} \) \quad [2]

\( \Rightarrow \) the series converges absolutely on \( -\frac{\sqrt{3}}{2} < x < \frac{\sqrt{3}}{2} \) \quad [2]

At \( x = \frac{\sqrt{3}}{2} \), the series is:

\[ \sum_{n \in \mathbb{N}} \frac{2^n}{n3^n} \left( \frac{3}{2} \right)^{2n} = \sum_{n \in \mathbb{N}} \frac{1}{n} = \infty \quad [2+2+2] \]

At \( x = -\frac{\sqrt{3}}{2} \), the series is:

\[ \sum_{n \in \mathbb{N}} \frac{2^n}{n3^n} \left( -\frac{3}{2} \right)^{2n} = \sum_{n \in \mathbb{N}} \frac{1}{n} = \infty. \quad [2+2+3] \]