READ CAREFULLY AND WORK ON ALL PROBLEMS. Justify your answers. Show all your steps. Cross out what is not part of your final answer. NO calculators or textbooks or pre-prepared notes are allowed. Total regular time: 50min.

1. (10 pts) Consider the vector field $\mathbf{F}$ defined by

$$\mathbf{F}(x, y, z) = yz^2 \mathbf{i} + xz^2 \mathbf{j} + [2xyz + 2\cos(2z)] \mathbf{k}.$$ 

(a)[6 pts] Compute $\text{curl} \mathbf{F}$. Determine a function $f(x, y, z)$ such that $\mathbf{F}(x, y, z) = \text{grad} f(x, y, z)$, if such a function $f$ exists. **Note:** For full credit, show all your steps in detail. Simply “guessing” this $f$ is NOT adequate.

(b)[4 pts] Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $C$ is the curve parametrized by

$$\mathbf{r}(t) = \cos(\pi t^9) \mathbf{i} + t^{10} \mathbf{j} + \frac{\pi t}{2} \frac{1}{1+t^4} \mathbf{k}, \quad -1 \leq t \leq 1.$$

2. (10 pts) Consider the line integral

$$I = \int_C y^4 dx + x^3 dy,$$

where $C$ is the oriented counterclockwise square in the $xy$-plane with vertices at the points $(-1, -1), (-1, 1), (1, -1), (1, 1)$.

By using Green’s theorem, write $I$ as a double integral over a suitable region $R$ of the $xy$-plane. Evaluate $I$. **Note:** You should compute $I$ by expressing it as an iterated integral.

3. (10 pts) By using Stokes’ theorem, evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F} = 3yi + 2zj - xk$ and $C$ is the triangle with vertices $(1, 0, 0), (0, 1, 0), (0, 0, 1)$ oriented counterclockwise as viewed from above. **Hint:** $C$ lies on the plane $x + y + z = 1$.

4. (10 pts) Let $D$ be the solid region bounded by the cylinder $x^2 + y^2 = 1$ and the planes $z = 1$ and $z = 2$. Let $\Sigma$ be the boundary of $D$. By using the Divergence Theorem, evaluate the flux integral $\iint_{\Sigma} \mathbf{F} \cdot \mathbf{n} dS$, where $\mathbf{F} = 2x \mathbf{i} + 3y \mathbf{j} + 5z \mathbf{k}$ and $\mathbf{n}$ is the unit normal vector pointing outward.