1. [10pts] Consider the investment cash flow sequence $C_1, C_2, \ldots, C_n$. Each amount $C_i$ is invested in the beginning of Year $i$ ($i = 1, 2, \ldots, n$). The interest rate is $r$, compounded yearly.

(a) [2pts] Define the Present Value, $P(r)$, of this cash flow sequence for a given rate $r$.

(b) [8pts] Assume that $C_i < 0$ for $i < n$, and $C_n > 0$. Show that there is a unique rate of return $r^*$ for this cash flow, i.e., show that there is a unique solution, $r = r^*$, of $P(r) = 0$ where $r > -1$.

2. [15pts] Consider two put options on the same security, both of which have expiration time $t$. Let the exercise prices of these put options be $K_1$ and $K_2$ where $K_1 < K_2$. Suppose that there is no arbitrage opportunity.

Prove that $K_2 - K_1 \geq P_2 - P_1$ where $P_i$ is the price of the put option with strike price $K_i$ ($i = 1, 2$). You may assume that the present value of $K_i$ is $\alpha K_i$ where $0 < \alpha \leq 1$ (and $\alpha$ depends on time $t$ but is independent of $K_i$). Caution: For full credit, do not ignore the influence of the costs of call option(s) on the same security.

3. [10pts] Assume that a game has the outcomes labeled 1, 2, 3. A wager on this game can be described as follows: One wins amount $o_i = 1, 2$ or 5 per unit bet if outcome $i = 1, 2$ or 3 happens, respectively; and loses amount 1 per unit bet otherwise. (The $o_i$’s describe the odds of this game.)

(a) [4pts] Find a betting strategy $(x_1, x_2, x_3)$ such that the return for each outcome is zero. Note: You only need to find one value for each $x_i$ ($i = 1, 2, 3$).

(b) [4pts] Suppose that $(y_1, y_2, y_3)$ is any strategy, and the returns are $(R_1, R_2, R_3)$; $R_j$ is the return of outcome $j$. Show that $3R_1 + 2R_2 + R_3 = 0$ regardless of the values of $y_1, y_2$ and $y_3$.

(c) [2pts] By using part 3(b), argue carefully that no arbitrage can occur in this game.

4. [15pts] The price of a security changes from one period to another period in such a manner that the price during period $i$ is the price during period $i - 1$ multiplied either by the factor $u$ ($u > 1$) or the factor $d = 1/u$, where $i \geq 1$. Assume that the price of the security in period 0 is $S$ ($S > 0$). The interest rate is $r$, compounded periodically.

Determine the no-arbitrage cost of a call option to purchase the security at the end of the first three periods (not counting period 0) for a strike price equal to $K$ where $dS < K < uS$.

Note: For full credit, simplify your answer as much as possible. Explain all your steps clearly and in detail.