Instructions: Answer each of the 11 numbered problems on a separate answer sheet. Each answer sheet must have your name, your TA’s name, and the problem number (=page number). Show all your work for each problem clearly on the answer sheet for that problem. You must show enough written work to justify your answers.

NO CALCULATORS

1. (10 points EACH)
   a) Compute \( f'(e) \) when \( f(x) = x^{-x} \).
   b) Compute \( \int_{1}^{2} (\ln x)^2 \, dx \).
   c) Evaluate \( \int \frac{1}{x^2 + 2x - 3} \, dx \).

2. (10 points EACH) Let \( f(x) = 3x \ln x \).
   a) Find the smallest number \( a \) so that the restriction of \( f \) to \((a, \infty)\) has an inverse.
      Let \( h \) denote the inverse of \( f \) restricted to this interval.
   b) Compute \( h'(0) \), where \( h \) is defined in part (a).

3. (10 points EACH)
   a) Find the sum of the series \( \sum_{n=0}^{\infty} \frac{(-2)^3}{3^{2n+1}} \).
   b) Does the series \( \sum_{n=1}^{\infty} \frac{\sin^2 n}{n^2} \) converge or diverge? EXPLAIN your answer.

4. (15 points) Find the radius of convergence of the series \( \sum_{n=2}^{\infty} \frac{2^n x^n}{(\ln n)^2} \).

5. (15 points) Suppose that \( y \) is a function of \( x \), satisfies the differential equation
   \[ y' = \frac{xy}{x + 2}, \text{ and } y(-1) = 1. \]
   Compute \( y(0) \).

Exam continues on reverse ----->
6. (10 points EACH) Recall that \( \frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \) for certain values of \( x \).

a) Find a power series for the function \( f(x) = \frac{x}{1-4x} \)

b) Use the power series in (a) to compute \( f^{(6)}(0) \), the sixth derivative of \( f \) evaluated at 0.

7. (15 points) Find the length \( L \) of the parametrized curve given by the equations
\[
x = e^t \cos t, \quad y = e^t \sin t
\]
for \( 0 \leq t \leq 1 \).

8. (15 points) Let \( R \) be the region bounded below by the \( x \)-axis, on the left by the \( y \)-axis, and above by the graph of \( y = \sqrt{1-x} \). Find the vertical coordinate \( \bar{y} \) of the center of gravity of \( R \).

9. (15 points) Let \( R \) be the region inside the rectangle with vertices at \( (3, 1) \), \( (6, 1) \), \( (3, 3) \), and \( (6, 3) \). Find the volume \( V \) of the solid region formed by revolving \( R \) around the \( x \)-axis.

10. (15 points) Determine whether the improper integral \( \int_{0}^{3} \frac{2x}{(1-x^2)^2} \, dx \) converges or diverges, giving reasons. If it converges, then compute its value.

11. (10 points EACH)

a) Find the area \( A \) of one of the 4 'petals' of the polar graph \( r = \cos 2\theta \).

b) Evaluate \( \int \frac{1}{\sqrt{16 - 25x^2}} \, dx \).

END OF EXAM – GOOD LUCK!