1.(20) True or false. Give a brief reason.
   a) If $A$ is an $m \times n$ matrix with $m < n$ then the system of equations $Ax = 0$ always has a non-trivial solution.
   b) The determinant of any $11 \times 11$ matrix with $A = -A^t$ is zero.
      ($A^t$ is the transpose of $A$.)
   c) If $P_n$ is the space of polynomials of degree $\leq n$ then the function $T: P_n \rightarrow \mathbb{R}^2$ with $T(p) = (p'(0), p(0)^2)$ is a linear map.
   d) If $A$ is a $12 \times 20$ matrix and and for every $y \in \mathbb{R}^{12}$ there exists $x \in \mathbb{R}^{20}$ with $Ax = y$ then there exists 8 linearly independent solutions to the equation $Ax = 0$.
   e) The linear map $R: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ given by rotation about the origin by $\frac{\pi}{2}$ in the counterclockwise direction has matrix \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}.

2.(10) Let $f(x, y) = (\cos x + \sin y + x, x^2 + 2y)$. Use the tangent approximation to find $(x, y)$ so that $f(x, y) \sim (.98, .01)$. (Note: $f(0, 0) = (1, 0)$.)

3.(10) A bug is moving in $\mathbb{R}^3$ with position $x(t) = (4t, \sqrt{3}t^2, t^3 + 3t)$ for $t \geq 0$.
   a) How far has the bug travelled (along the path) after 1 second.
   b) If $T(x, y, z) = x^2 + 2y^2 - z^2$ is the temperature at a point $(x, y, z)$ is the bug getting warmer or cooler at $t = 1$ if he continues along the path?

4.(5) Let $\Sigma$ be intersection of the ellipsoid $2x^2 + y^2 + z^2 = 2$ and the smooth surface $\sin(xy) + yz = 1$. Explain why in a neighborhood of $(0, 1, 1) \in \Sigma$, $(x, y)$ may be written as functions of $z$ but it is not clear if $(y, z)$ may be expressed in terms of $x$.

5.(5) Explain why if $f: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a differentiable function and $f(0, 0) = (1, 1)$ and $Df(0, 0) = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ then there does not exist a differentiable function $g: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ with $(f \circ g)(x, y) = (y, x)$ and $g(1, 1) = (0, 0)$.

6.(10) Compute the following integrals:  
   a) $\int_0^\pi \int_y^\pi \frac{\sin(x)}{x} \, dx \, dy$. 

b) \( \int_{R} (x^2 - y^2) \, dA \) where \( R \) is the region in the \( xy \)-plane bounded by the lines \( x + y = 1 \) and \( x + y = 2 \), \( y = x \) and the \( y \)-axis.

7. (10) Let \( C \) be a closed curve in \( \mathbb{R}^2 \). Use Green’s theorem to show the two vector fields \( \mathbf{F}_1(x, y) = (0, x) \) and \( \mathbf{F}_2(x, y) = (y, 0) \) do opposite work around the closed curve \( C \).

8. (10) Find the surface area of that part of the paraboloid \( z = \frac{1}{2}(x^2 + y^2) \) cut out by the cylinder \( x^2 + y^2 = 4 \).

9. (10) Let \( \mathbf{F}(x, y, z) = (0, 0, z^2) \) and let \( D \) be the solid region in \( \mathbb{R}^3 \) given by \( \{(x, y, z) \mid 0 \leq x^2 + y^2 \leq 1, 0 \leq z \leq 1\} \). Verify Gauss’s Theorem by computing both sides of the equation.

10. (10) Let \( \mathbf{F}(x, y, z) = (y^2, x^2 + z, -xy) \). Compute line integral \( \int_{\Sigma} \mathbf{F} \, d\mathbf{x} \) where \( \Sigma \) is the intersection of the paraboloid \( z = (x - 1)^2 + (y - 1)^2 \) and the plane \( 2x + 2y + z = 6 \) oriented counterclockwise when viewed from above.