(1) Suppose \( f(x, y, z) \) is a twice differentiable function. Show that \( \text{curl}(\nabla f) = 0 \).

(b) Suppose \( \mathbf{F}(x, y, z) \) is a (continuously differentiable) vector field defined in an open set \( S \) in \( \mathbb{R}^3 \), and \( \text{curl}(\mathbf{F}) = 0 \). Is there necessarily a function \( f(x, y, z) \) such that \( \nabla f = \mathbf{F} \) in \( S \)? Justify your answer.

(2) Suppose \( \alpha > 0 \) is a constant, and
\[
\mathbf{F}_\alpha(x, y) = \left( \frac{-y}{(x^2 + y^2)^\alpha}, \frac{x}{(x^2 + y^2)^\alpha} \right)
\]

(a) Compute \( \int_\gamma \mathbf{F}_\alpha \cdot \mathbf{dx} \) where \( \gamma \) is the circle of radius \( R \), centered at the origin, traced counterclockwise.

(b) Compute the scalar curl of \( \mathbf{F} \), and show that it is 0 if and only if \( \alpha = 1 \).

(c) Take \( \alpha = 1 \). Let \( \gamma \) be the circle, centered at a point \( (x_0, y_0) \), with radius \( R \neq \sqrt{x_0^2 + y_0^2} \), traced counter-clockwise. What is \( \int_\gamma \mathbf{F}_\alpha \cdot \mathbf{dx} \)? Your answer will depend on \( (x_0, y_0) \) and \( R \). Justify your answer.

(3) Find the general (real) solution of
\[
y'' - 2y' + 10y = 0, \quad y(0) = 2, \quad y'(0) = 3
\]
What happens to the solution as \( x \to \infty \)?

(4) Find the general (real) solution of
\[
y'' - 5y' + 4y = e^x
\]

(5) Solve
\[
y' + \sin(3x)y^2 = 0, \quad y(0) = 1
\]
What is \( y(\pi) \)?