1. page 280, #1. After the substitution you have \( \int_C \frac{1}{2z^2 + 5iz - 2} \, dz \). The roots of the denominator are \(-\frac{1}{2}i, -2i\), and only \(-\frac{1}{2}i\) is inside the contour. The residue is \( \frac{1}{3i} \), and the solutions is \( 2i \cdot \frac{1}{3i} = \frac{2}{3}i \).

2. #2 The standard substitution gives \( -4i \int_C \frac{z}{z^4 - 6z^2 + 1} \, dz \). The denominator has roots \( z^2 = 3 \pm 2\sqrt{2} \), or \( z = \pm \sqrt{3 \pm 2\sqrt{2}} \). Only \( \sqrt{3 - 2\sqrt{2}} \) is inside the contour. Note that \( \sqrt{3 - 2\sqrt{2}} = 1 - \sqrt{2} \).

These are simple poles. Use Theorem 2, page 243 to evaluate the residues. The derivative of \( \frac{z}{z^4 - 6z^2 + 1} \) is \( \frac{1 - \sqrt{2}}{4(z - \sqrt{2})(1 - \sqrt{2})^2 - 3} \). At \( -1 + \sqrt{2} \) the residue is the same. So the solution is \( 2\pi i (\sqrt{2}/8) = 2\sqrt{2} \).

Here is a clever trick due to one of the students: use the substitution \( z = e^{2i\theta} \). Then \( dz = \frac{dz}{2iz} \). Also \( \sin^2(\theta) = \frac{1}{2}(1 - \cos(2\theta)) = \frac{1}{2}(1 - \frac{z + 1}{2}) = \frac{2 - z + 1}{4} \). The integral becomes \( 4i \int_C \frac{1}{z^2 - 6z + 1} \, dz \). Notice this is a quadratic in the denominator, and therefore much easier to compute.

3. #7 I discussed this in class (use the binomial theorem).

4. page 296, #1 The poles are at \( s = \pm \sqrt{2}, \pm i\sqrt{2} \). The residues at \( \pm \sqrt{2} \) are \( \frac{\xi_{\pm \sqrt{2}}}{2} \), and at \( \pm i\sqrt{2} \) are \( \frac{i\xi_{\pm \sqrt{2}}}{2} \). Adding these we get \( \cos(\sqrt{2}t) + \cosh(\sqrt{2}t) \).

5. #4. The poles are at \( \pm ai \). Use the technique of Example 1, page 292. Write \( \frac{s^2 - a^2}{(s^2 + a^2)^2} = \frac{(s^2 - a^2)((s + ai)^2)}{(s - ai)^2} = \phi(s)/(s - ai)^2 \). The sum of the residues of at \( \pm ai \) is \( 2Re[(e^{at}(b_1 + b_2t)) \] where \( \frac{s^2 - a^2}{(s^2 + a^2)^2} = \frac{b_1}{s - ai} + \frac{b_2t}{(s - ai)^2} + \ldots \). Then \( b_1 = \phi'(ai) = 0 \), and \( b_2 = \phi(ai) = \frac{1}{2} \).

This gives \( 2Re(e^{at}(\frac{1}{2}i)) = t \cos(at) \).