1. (May, top of page 59) Let \( p: E \to B \) be a fibration, and suppose \( E \) and \( B \) are equipped with basepoints and \( p \) sends basepoint to basepoint. Show that \( p \) has the *based* homotopy lifting property for based maps \( Y \to B \), where \( Y \) is a based space with non-degenerate basepoint \( y_0 \) (i.e., such that \( \{y_0\} \hookrightarrow Y \) is a cofibration).

2. The unitary group is the compact group \( U(n) \) of \( n \times n \) complex-valued matrices \( u \) such that \( u^* = u^{-1} \). (Here \( u^* \) is the adjoint with respect to the usual inner product on \( \mathbb{C}^n \), i.e., the conjugate transpose of \( u \).)

   1) Show that \( U(n) \) operates transitively on the unit sphere \( S^{2n-1} \) in \( \mathbb{C}^n \) (by matrix multiplication, when we think of \( \mathbb{C}^n \) as consisting of column vectors), and that the isotropy group of \( (0, \ldots, 0, 1)^t \) can be identified with \( U(n-1) \). Here \( U(n-1) \leq U(n) \) via \( u \mapsto \begin{pmatrix} u & 0 \\ 0 & 1 \end{pmatrix} \). Then show that the map \( p: U(n) \to S^{2n-1} \) defined by \( u \mapsto u \cdot (0, \ldots, 0, 1)^t \) is a fiber bundle (and hence a fibration) with fiber \( U(n-1) \).

   2) Deduce that there is an exact sequence of homotopy groups

   \[ \cdots \to \pi_j(U(n-1)) \to \pi_j(U(n)) \to \pi_j(S^{2n-1}) \to \cdots \]

   and thus that \( \pi_j(U(n-1)) \to \pi_j(U(n)) \) is an isomorphism for \( j < 2n - 2 \).

   3) Show that the map \( \pi_j(U(n-1)) \to \pi_j(U(n)) \) is not necessarily an isomorphism for \( j = 2n - 1 \). (Hint: what is \( U(1) \)?

   4) (A little harder than (3)) Show from the same exact sequence that \( \pi_j(U(n)) \) contains a summand of \( \mathbb{Z} \) when \( n \geq (j+1)/2 \) and \( j \) is odd.