1. (12 points)
   (a) (10 pts) Let \( S(x) = 8 \ln(7x) - 3x \) define a “satisfaction function” for \( x \) in \([2, 5]\). Find the value of \( x \) at which \( S \) achieves a maximum.

   (b) (2 pts) Briefly explain how you know the maximum is achieved at this \( x \).

2. (13 points)
   (a) (9 pts) Find the equation of the tangent line to the curve
   \[ xy^3 + \ln(y) = x^2 - 6 \]
   at the point \((x, y) = (3, 1)\).

   (b) (4 pts) The top half of the circle \( x^2 + y^2 = 25 \) is the graph of the function
   \( y = \sqrt{25 - x^2} \).
   What is \( \int_{-5}^{5} \sqrt{25 - x^2} \, dx \)?

3. (10 points)
   Find the area of the region bounded between the two curves \( y = x^3 \) and \( y = x^2 \).

4. (13 points) Evaluate the following definite integrals.
   \[ (a) \ (6 \text{ pts}) \int_{0}^{1} x^2 \sqrt{x^3 + 1} \, dx \quad (b) \ (7 \text{ pts}) \int_{1}^{3} \frac{\ln x}{x} \, dx \]

**** THERE ARE MORE PROBLEMS ON THE OTHER SIDE. ****
5. (12 points)
Evaluate the following indefinite integrals.

(a) (6 pts) \( \int \frac{1}{(5x + 2)^2} \, dx \)  
(b) (6 pts) \( \int \tan(x) \, dx \)

6. (10 points)
At time \( t \) days, the rate at which a certain substance grows is \( 200 e^{0.1t} \) milligrams per day.
What is the total accumulated growth of the substance over the time interval \( 0 \leq t \leq 3 \) ?

7. (15 points) A 13-foot ladder is placed against a building. The base of the ladder is slipping away from the building at a rate of 4 feet per minute.
Find the rate at which the top of the ladder is sliding down the building at the instant when the bottom of the latter is 5 feet from the base of the building.

8. (15 points) A fence must be built to enclose a rectangular area of 5,000 square feet. Fencing material costs \$2.50\) per foot for the two sides facing north and south, and it costs \$3.20 per foot for the other two sides.
For the rectangle of area 5,000 square feet which is enclosed by the cheapest possible fence, what is the length of the side facing north?