2. \( \left( \frac{3}{p} \right) = (-1)^{\frac{p-1}{2}} \left( \frac{3}{p} \right) \). Both factors are 1 if \( p \equiv 1 \) (mod 12) and both are -1 if \( p \equiv -1 \) (mod 12). If \( p \equiv \pm5 \) (mod 12), the two factors have opposite signs so that the product is -1.

4. \( \left( \frac{p}{3} \right) = \left( \frac{p}{5} \right) = 1 \) if and only if \( p \equiv 1, 4 \) (mod 5).

6. Let \( Q = 5(n!)^2 - 1 \). Then 5 and \( n! \) are both relatively prime to \( Q \). Let \( x \) be a modular inverse for \( n! \) (mod \( Q \)). Then \( x^2 \equiv 5 \) (mod \( Q \)). It follows that 5 is a quadratic residue (mod \( Q \)) and hence (mod \( p \)) for every prime divisor \( p \) of \( Q \). Hence by problem 4, every prime divisor of \( Q \) is congruent to 1 or to 4 (mod 5). But \( Q \equiv 4 \) (mod 5). Hence all the prime divisors of \( Q \) cannot be congruent to 1 (mod 5), and at least one of them must be congruent to 4 (mod 5) as desired. Moreover, all prime divisors of \( Q \) are relatively prime to \( n! \), and hence larger than \( n \). It follows that there are arbitrarily large primes congruent to 4 (mod 5), and hence there are infinitely many of them.