1) Prove by contradiction that \( \sqrt{3} \) is irrational.

2) Prove that the product of a nonzero rational and an irrational is irrational.

3) a) Disprove: If \( a, b \in \mathbb{R} \), then \( \log(a + b) = \log a + \log b \).
   b) Prove there exists a real solution to \( x^3 + x - 1 = 0 \) between \( x = 0 \) and \( x = 1 \).
   c) Disprove: There exists a real number \( x \) such that \( x^4 + 3x^2 + 1 = 0 \).

4) Prove that if \( x, y \in \mathbb{R} \), then \( \sqrt{x + y} \neq \sqrt{x} + \sqrt{y} \).

5) Let \( n \) be a positive integer of the form \( n = 2r \) where \( r \) is odd. Prove that there do not exist integers \( x \) and \( y \) such that \( x^2 - y^2 = m \).

6) Prove by induction that \( 1 + 3 + 5 + \cdots + (2n - 1) = n^2 \) for all \( n \in \mathbb{N} \).

7) Let \( s \neq 1 \) be a real number. Use induction to prove that \( b + bs + bs^2 + bs^3 + \cdots + bs^{n-1} = \frac{b(1-s^n)}{1-s} \) for all \( n \in \mathbb{N} \).

8) Prove for all integers \( n \geq 0 \) that \( 4|(5^n - 1) \).

9) Prove for all integers \( n \geq 4 \) that \( 3^n > n^3 \).