1 (15 pts). Problem 1.8 in Moler.

2 (20 pts). Problems 1.34 and 1.35 in Moler.

3 (15 pts). Problem 1.38 in Moler.


5 (15 pts). The first derivative of the function $f$ can be approximated by the second order formula

$$d(h) = \frac{f(a + h) - f(a - h)}{2h}$$

(a) Use Taylor expansion to show that the error $e(h)$ in its approximation can be bounded by

$$|e(h)| \leq \frac{M}{6} h^2,$$

where $M$ is a bound on the 3rd derivative of $f$.

(b) Write a MATLAB function

```matlab
function d = Derivative(fname,a,n)
```

that approximates the first derivative of the function `fname` with the above expression for $h = 10^{-k}$ with $k = 1 : n$. Use the command `eval` to evaluate `fname`. To invoke the function with `fname = f(x)` use the syntax `d = Derivative('f',a,n)`.

(c) Use this function to approximate the 1st derivative of $f(x) = \sin 5x$ at $a = 1$ with $n = 16$. Print the vector $d$ and the actual error $|e(h)|$.

(d) Explain why $e(h)$ does not behave as expected. To this end use (a) and approximate the roundoff error by $|f'(a)| \frac{\epsilon}{h}$, where $\epsilon$ is the machine epsilon.

6 (20 pts). The numbers $p_n = \int_0^1 x^n e^x dx$ satisfy $p_1 > p_2 > \cdots > 0$ and the recursion relation

$$p_{n+1} = e - (n + 1)p_n, \quad p_1 = 1.$$

(a) Prove the recurrence relation.

(b) Write a MATLAB program to generate the first 20 values of $p_n$ and explain why the inequalities above are violated (do not use subscripted variables).

(c) Let $p_{20} = 1/8$ and use the recurrence relation to compute $p_{19}, \cdots, p_1$. Do the numbers satisfy the inequalities $1 = p_1 > p_2 > \cdots > 0$? Explain the difference in the two procedures in terms of numerical stability. Repeat with $p_{20} = 20$ and $p_{20} = 100$. Explain what happens.