1 (25 pts). **Multistep Method:** Suppose that two initial guesses \( y_0, y_1 \) are known for the nonlinear ODE
\[
y' = f(t, y).
\]
Let \( h > 0 \) be the constant stepsize. The 2nd order Gear’s formula is a 2-step method that computes \( y_{n+1} \) according to the formula
\[
y_{n+1} = \frac{4}{3}y_n - \frac{1}{3}y_{n-1} + \frac{2}{3}hf(t_{n+1}, y_{n+1}), \quad (n \geq 1).
\]
Interpolate the values \((t_i, y_i)\) for \( i = n-1, n, n+1 \) with a quadratic polynomial \( q(t) \) and next set \( q'(t_{n+1}) = f(t_{n+1}, y_{n+1}) \) to deduce the formula above. To do this, use the Lagrange form of \( q(t) \). The Gear’s formulas are implicit, and are effective for **stiff** problems.

2 (25 pts). Problem 5.8 of Moler. In addition do part (a) with the **normal equations**.

3. Converting a matrix \( A \in \mathbb{R}^{n \times n} \) into an **upper bidiagonal** matrix \( B \in \mathbb{R}^{n \times n} \), namely the diagonal and upper diagonal are the only nonzero entries of \( B \), is a useful process in eigenvalue computations. It is used to compute the **singular value decomposition** of \( A \).

   (a) (15 pts) Study how to use Householder reflections, in the spirit of the QR factorization, to produce a sequence of orthogonal matrices \( H_1, H_2, \ldots, H_{2(n-1)-2}, H_{2(n-1)-1} \in \mathbb{R}^{n \times n} \) such that
   \[
   H_{2(n-1)-1} \cdots H_3 H_1 A H_2 H_4 \cdots H_{2(n-1)-2} = B.
   \]
   The only difference with QR is that every other time the Householder matrix multiplies from the right instead of the left. Notice that the subscripts indicate the order in which the Householder matrices are applied. Hint: examine first \( n = 3 \) and argue with transposition to examine right multiplication.

   (b) (10 pts) Perform an operation count. Note that you are not required to compute and store the \( H_i \)'s. Hint: the action of a rank-1 matrix \( uv^\top \) on a vector \( x \), namely \((uv^\top)x\), is better computed as follows: \((v^\top x)u\).