1. (a) (4 points) 13.4 #30.

Differentiate by x to obtain $0 = 2xz^2 + x^22z\frac{\partial z}{\partial x} - 2yz - 2xy\frac{\partial z}{\partial x} + 3z^2y^2\frac{\partial z}{\partial x}$

$$\frac{\partial z}{\partial x} = \frac{2yz - 2xz^2}{3z^2y^2 - 2xy + 2x^2z}$$

Similarly, $\frac{\partial z}{\partial y} = \frac{2xz - 2z^3y}{2x^2z - 2xy + 3z^2y^2}$

(b) (4 points) 13.4 #34.

Let $w = f(r, s, t)$ where $r = x - y; s = y - z; t = z - x$.

$$\frac{\partial w}{\partial x} = \frac{\partial w}{\partial r} \frac{\partial r}{\partial x} + \frac{\partial w}{\partial s} \frac{\partial s}{\partial x} + \frac{\partial w}{\partial t} \frac{\partial t}{\partial x} = \frac{\partial w}{\partial r} - \frac{\partial w}{\partial t}$$

In a similar way, we compute:

$$\frac{\partial w}{\partial y} = \frac{\partial w}{\partial s} - \frac{\partial w}{\partial r} \quad \text{and} \quad \frac{\partial w}{\partial z} = \frac{\partial w}{\partial t} - \frac{\partial w}{\partial s}$$

With this we compute $\frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z} = 0$

(c) (4 points) 13.4 #38.

$f(tx, ty) = t^n f(x, y)$ Write $r = tx$ and $s = ty$

$$\frac{\partial f}{\partial t}(r, s) = x \frac{\partial f}{\partial r} + y \frac{\partial f}{\partial s} = nt^{n-1}f(x, y)$$

Putting $t = 1$ gives $xf_x(x, y) + yf_y(x, y) = nf(x, y)$
2. (8 points) 13.5 #16.

\[ \nabla f(x, y) = (w + wxy, x^2 + \sin(y) + \cos(y)) \]

a) \( D_\vec{u} f(1, 0) = \nabla f(1, 0) \cdot \vec{u} = (2, 1) \cdot (a, b) = 2a + b \)
(subject to \( a^2 + b^2 = 1 \)).

b) \( D_\vec{u} f(1, 0) \) is maximal when \((a, b)\) is parallel to \( \nabla f(1, 0) \).
That is, when \((a, b) = \left( \frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}} \right) \)

3. (a) (5 points) 13.6 #62.

\[ z = 5 - x^2 - 2y^2 = f(x, y) \text{ is the altitude at } (x, y). \]

\[ \nabla f(x, y) = (-2x, -4y) \]

\[ \nabla f(1/2, -1/2) = (-1, 2) \]

(b) (5 points) 13.6 #64. (Use the back if necessary.)