Problems to be handed in:

2. Braun 3.11, # 4, 7, 17.
3. Braun 3.12, # 2, 3, 5, 6, 8-10.

4. Theorem. Every $n \times n$ matrix with complex entries can be put into Jordan normal form.

More precisely, if $A \in M_n(\mathbb{C})$, then there exists an invertible $n \times n$ complex matrix $P$ such that $P^{-1}AP$ is in Jordan normal form, that is, it consists of block matrices down the diagonal, each block (called a Jordan block) being of form

$$
\begin{pmatrix}
\lambda & 1 & \cdots & \cdots & 0 \\
0 & \lambda & 1 & \cdots & 0 \\
0 & & \ddots & & \\
0 & & \cdots & \lambda & 1 \\
0 & & \cdots & \cdots & \lambda
\end{pmatrix}.
$$

(Aside from the block matrices down the diagonal, all the other entries are zero.) For example, the following matrix is in Jordan normal form:

$$
\begin{pmatrix}
2 & 1 & 0 & 0 & 0 \\
0 & 2 & 0 & 0 & 0 \\
0 & 0 & 2 & 1 & 0 \\
0 & 0 & 0 & 2 & 0 \\
0 & 0 & 0 & 0 & 3
\end{pmatrix}
$$

and it has three Jordan blocks in it.

Clearly the entries down the diagonals in the various Jordan blocks in $P^{-1}AP$ are nothing other than the eigenvalues of $A$.

Suppose that given $A$, you know a matrix $P$ such that $P^{-1}AP$ is in Jordan normal form. Explain how you can use this to completely solve the differential equation $\frac{dx}{dt} = Ax$. Hint: Can you compute explicitly $e^{P^{-1}APt}$?