Homework 1 – due 10/06/05
Math 603

Do as many as you can!

1. Atiyah-Macdonald, Exercise 21, Chapter 1.


3. Let $k$ be any algebraically closed field, and let $B$ be a domain which is a finitely generated $k$-algebra. Let $x$ be a closed point of $\text{Spec}(B)$, and let $U \subset \text{Spec}(B)$ be a non-empty open set. Show that there is a curve in $\text{Spec}(B)$ joining $x$ to some point in $U$. (Note: for the purposes of this exercise, a curve in $\text{Spec}(B)$ is a closed set of form $V(p), p$ a prime ideal, where dim$(B/p) = 1$. Also, to avoid trivial counterexamples, we need to assume $B$ is not a field.) Hint: do it first in the case $B = k[X_1, \ldots, X_n]$, then reduce to this case by using the Noether Normalization and the Going-Down theorem.

4. Exercise 4.3.3 from the notes.

5. Exercise 6.3.3 from the notes.

6. Exercise 7.6.2 from the notes.