Vocabulary from 10.2: A matrix with the same number of rows as columns is a **square matrix**, for example
\[
\begin{bmatrix}
1 & -2 & 0 \\
0 & 1 & -1 \\
2 & -3 & 2
\end{bmatrix}
\] is a 3 × 3 square matrix and
\[
\begin{bmatrix}
3 & -1 \\
1 & -2
\end{bmatrix}
\] is a 2 × 2 square matrix.

Now a new definition: The **identity matrix** \(I_n\) is a square matrix such that, for any \(n \times n\) square matrix \(A\), it will be true that \(AI = IA = A\).

An identity matrix will necessarily be a square matrix. Why? Recall \(NP\) vs. \(PN\) from Lecture 10.3. For non-square matrices, \(AI\) would be a different size than \(IA\), so they could not be equal.

Our goal now is to find, if possible, a **multiplicative inverse** matrix \(A^{-1}\) such that, for an \(n \times n\) square matrix \(A\), it will be true that \(AA^{-1} = A^{-1}A = I_n\). Note: Not all square matrices have an inverse!

An identity matrix \(I_n\) will necessarily have diagonal entries = 1 and upper and lower triangle entries = 0. (Sound familiar?)

\[
I_3 = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

\[
I_2 = \begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
\]

Example A:

Given \(B = \begin{bmatrix}
1 & -2 & 0 \\
0 & 1 & -1 \\
2 & -3 & 2
\end{bmatrix}\), find \(B^{-1}\) (if possible). \textit{Answer:} \(B^{-1} = \begin{bmatrix}
-\frac{1}{3} & \frac{4}{3} & \frac{2}{3} \\
-\frac{2}{3} & \frac{2}{3} & \frac{1}{3} \\
-\frac{2}{3} & -\frac{1}{3} & \frac{1}{3}
\end{bmatrix}\)

I’ll leave it to you to check that check that \(BB^{-1} = I_3\), as practice in matrix times matrix multiplication.
Example A continued: (You’ll probably need the space below.)

Example B: Given \( C = \begin{bmatrix} 6 & 2 \\ 12 & 4 \end{bmatrix} \), find \( C^{-1} \) (if possible). \textit{answer:} \( C \) has no inverse.

Your text derives a formula for the inverse of a \( 2 \times 2 \) square matrix: \( A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \), \( A^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \).

I usually don’t recommend memorizing this, because it can go wrong so very easily. Note, however, that it does tell us which \( 2 \times 2 \) matrices don’t have an inverse. When \( ad - bc = 0 \), the matrix does not have an inverse. The formula \( ad - bc \) is called the \textbf{determinant} of the \( 2 \times 2 \) matrix \( A \).
Theory for solving matrix equations: \( AX = B \implies A^{-1}AX = A^{-1}B \implies IX = A^{-1}B \implies X = A^{-1}B. \)

(See your text for the detailed explanation.) Note that it must be \( X = A^{-1}B \). Don’t do \( BA^{-1} \), which is not the same thing at all: matrix times matrix multiplication is not commutative!

Example C: Solve the system of equations

\[
\begin{align*}
-2x + 3y &= 5 \\
3x - y &= -5
\end{align*}
\]

using the inverse of a matrix.

answer: \( x = -\frac{10}{11}, \quad y = \frac{25}{11} \)