(1) Let $\lambda \in \mathfrak{h}^*$. Show that the set $\Phi[\lambda] := \{\alpha \in \Phi; \langle \lambda, \alpha^\vee \rangle \in \mathbb{Z}\}$ is a root system.

(2) Let $\mathfrak{g} = \text{so}(5)$. Find $\lambda \in \mathfrak{h}^*$ such that $\Phi[\lambda]$ is a root system of type $A_1 \times A_1$.

(3) Let $s$ be a simple reflection and $\mu$ be an antidominant integral weight that lies only in the $s$-wall, i.e., $\langle \mu + \rho, \alpha_s^\vee \rangle = 0$ and $\langle \mu + \rho, \alpha_t^\vee \rangle < 0$ for all the simple reflections $t \neq s$. Set $\theta_s = T_{\mu - 2\rho}^T T_{-2\rho}^\mu$. Show that in the Grothendieck group of $\mathcal{O}_0$, $[\theta_s \Delta(w \cdot 0)] = [\Delta(w \cdot 0)] + [\Delta(ws \cdot 0)]$.

(4) Let $w_0$ be the longest element in the finite Weyl group $W$. Prove that $H_w = \sum_{y \in W} r_{\ell(w_0) - \ell(y)} H_y$.

(5) Let $r_{y,w} \in \mathbb{Z}[v^{\pm 1}]$ with $H_w = \sum \bar{r}_{y,w} H_y$. Prove that

(i) $r_{y,w} = r_{ww_0,yw_0} = r_{w_0w_0,yw_0}$.

(ii) $\bar{r}_{y,w} = (-1)^{\ell(y) + \ell(w)} r_{y,w}$.