REVIEW OF "TOPOLOGY AND GEOMETRY" BY GLEN E. BREDON

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April 21, 1995

The book under review is an excellent textbook suitable for a two-year course sequence in algebraic and differential topology. It has been used here at the University of Maryland for the basic sequence in geometry/topology for two years. I look forward to teaching from it next year.

This ambitious book covers basic algebraic topology (homotopy theory and homology theory), differential topology (vector bundles, tubular neighborhoods and transversality) and Lie groups. Although the book covers the Gysin and Wang exact sequences and the Steenrod cohomology operations, it doesn't get into spectral sequences. Similarly it treats de Rham theory and differential forms, but it doesn't get to harmonic theory or Morse theory. It can't cover everything, of course, and being 557 pages long, it covers an amazing amount of material thoroughly. The book is full of exercises of varying difficulty and contains a complete index.

The book starts with a discussion of general topology. This is not a comprehensive treatment of the subject, but I feel the author devotes the right amount of time to this, as most applications require more structure. The second chapter deals with differentiable manifolds, which I feel is quite appropriate, since so much topological intuition derives from examples of smooth manifolds. Furthermore many theorems have alternate proofs in the smooth case, which can be shorter (once the machinery is developed) and more intuitive than homological proofs. The difficulty, of course, is that much more theory must be developed to handle the additional structure. However, this material is so fundamental that students should see this now.

The book contains a rich supply of examples. The section on the fundamental group ends with a description of $SO(3)$, basic to many applications. The theory is illustrated by an experiment ("undoing a double twist"), which should be very useful to a student learning the abstract theory.

After singular homology is introduced, the Eilenberg-Steenrod axioms are stated. From this basic applications are derived, such as the Brouwer fixed point theorem and the nonexistence of nonsingular vector fields on even-dimensional spheres. Degree theory is discussed and leads to an extensive treatment of CW-complexes and cellular homology. This machinery permits many deep applications such as the generalized

Jordan curve theorem, invariance of domain, the generalized Schoenflies theorem. Simplicial complexes are only introduced here, and the chapter concludes with the the Lefschetz-Hopf fixed point theorem.

The next section treats cohomology, and starts right in with a discussion of exterior differential forms. Since most students at this level are at home with integration, probably a typical student feels more comfortable with differential forms than general singular cochains. After de Rham theory is developed, singular theory resumes. A whole section is devoted to complex projective space, certainly an important and valuable example. Explicit calculations with differential forms are given, as well as the relationship with its cell structure and the Hopf map. This nontrivial example illustrates many of the ideas developed earlier in the book. A reader willing to put in the effort to study this example carefully will benefit greatly. Hopf's theorem on the homotopy classification of self-maps of S^n is proved here (foreshadowing the obstruction theory later to come). The chapter concludes with a discussion (10 pages) of differential forms on compact Lie groups, certainly another one of the basic examples in geometry. Invariant differential forms are discussed, and the rank of $H^3(G)$ is identified with the number of simple factors of a compact semisimple Lie group G.

The next chapter, entitled "Products and duality," develops the standard theory of cross, cup, slant and cap products, with abundant examples to illustrate the abstract theory. Before discussing Poincaré duality from the algebraic viewpoint, a section on the "classical outlook" develops the more intuitive approach for triangulated manifolds. Applications include the calculation of the cohomology of complex projective spaces, Thom's theorem on the vanishing of the signature of a boundary, examples (lens spaces) of closed 3-manifolds with isomorphic homotopy and homology groups which are not homotopy-equivalent. The next section discusses intersection theory, the Euler and Stiefel-Whitney characteristic classes, as well as a thorough discussion of Lefschetz numbers. The standard applications (fundamental theorem of algebra, conjugacy of maximal tori in compact Lie groups) are given here. A treatment of the Steenrod squares is given here The chapter concludes with a discussion of plumbing. There is clearly a wealth of interesting mathematics discussed here.

Homotopy theory is discussed in the last chapter. Whitehead products, the Whitehead and Hurewicz theorems, and the exact homotopy sequence of a fibration are discussed. The homotopy group $\pi_{n+2}(S^n)$ is computed. The discussion of obstruction theory is particularly welcome, since I have often tried to direct graduate students, but most of the references with which I am familiar are not recent. It seems that the author included everything that could be done without spectral sequences, which is entirely appropriate, since a thorough treatment of this material would add quite a bit to an already enormous book.

Some of the students here who have read the book give it high marks for comprehensibility. The treatment of quotient spaces, in particular, received favorable comments. I heard comments like "it's all there" and "it's a great book to have." Rarely have I heard such favorable comments concerning a textbook. The section on paracompactness was too short for some, although I found it quite satisfactory. I didn't like the definition of properly discontinuous action (a personal pet peeve): the definition given here (which I am used to calling "wandering") implies that the quotient map is a branched covering, but doesn't imply the quotient is Hausdorff. (In order to assure the quotient space be Hausdorff, the action is usually assumed to be a *proper action*). However, this is a very small issue in light of the grand scope of this book and detracts very little.

The author has made a real contribution to the mathematical literature with this book. I would like to see more books of its level of scholarship and pedagogy available to students.

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