This is a review text file submitted electronically to MR.

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Short title: Variétés anti-de Sitter de dimension 3 possèdant un champ de Killing non trivial

Control number: 1 443 988

Primary classification: 53 C

Secondary classification(s):

Review text:

This note classifies all compact 3-dimensional Lorentz manifolds of constant negative curvature which possess a nontrivial Killing field. Such manifolds are locally isometric to *anti-de Sitter space* X, a convenient model of which is the universal covering group $\widetilde{\mathrm{PSL}(2,\mathbb{R})}$, with isometries given by the group $G = \widetilde{\mathrm{PSL}(2,\mathbb{R})} \times \widetilde{\mathrm{PSL}(2,\mathbb{R})}$ where one factor acts by left-multiplication and the other by right-multiplication. Kulkarni and Raymond (J. Diff. Geo. **21** (1985), 113–138) gave the first examples of compact anti-de Sitter manifolds. Their examples are quotients of anti-de Sitter space by discrete groups of the form graph(ρ), where $\rho : \Gamma \longrightarrow \widetilde{\mathrm{PSL}(2,\mathbb{R})}$ is a homomorphism taking values in a compact subgroup of $\widetilde{\mathrm{PSL}(2,\mathbb{R})}$, and $\Gamma \subset \widetilde{\mathrm{PSL}(2,\mathbb{R})}$ is a discrete group. (This condition is useful since it implies that the holonomy group lies in a subgroup $H \subset G$ which acts transitively and properly on X.) Furthermore, Kulkarni and Raymond showed that every properly discontinuous action with compact quotient is equivalent to the graph of a homomorphism $\rho : \Gamma \longrightarrow \widetilde{\mathrm{PSL}(2,\mathbb{R})}$ where $\Gamma \subset \widetilde{\mathrm{PSL}(2,\mathbb{R})}$ is a discrete cocompact subgroup.

Subsequent further examples were given by Goldman (J. Diff. Geo. **21** (1985), 301–308) and Ghys (Ann. Sci. Ècole. Norm. Sup. **20** (1987), 251–270) all of which possess nontrivial Killing vector fields, but the holononmy does not lie in a properly acting transitive subgroup of G. The difficulty in these examples is that in general compactness of an indefinite metric does not imply completeness.

Recently Klingler (Math. Ann. **306** (1996), 353–370) proved that compact anti-de Sitter manifolds are geodesically complete; thus any compact Lorentzian manifold of constant negative curvature is a quotient of anti-de Sitter space. It follows that whenever ρ is sufficiently close to the trivial representation, graph(ρ) acts properly and cocompactly on X. The paper under review, determines explicitly how small ρ must be for graph(ρ) to act properly, in the special case ρ takes values in an abelian subgroup of $\widetilde{\mathrm{PSL}(2,\mathbb{R})}$. If ρ takes values in an elliptic or parabolic one-parameter subgroup, then graph(ρ) acts properly. Thus, suppose that ρ takes values in a hyperbolic one-parameter subgroup H. For $\gamma \in \Gamma$, let $t(\gamma)$ the minimum displacement of γ acting on the hyperbolic plane. Then the stable norm on $H^1(\Gamma, \mathbb{R})$ is defined as the the function

$$\|h\| = \lim_{n\to\infty} \frac{1}{n} \inf\{t(\gamma_n) \mid [\gamma_n] = nh\}$$

where $[\gamma]$ denotes the equivalence class of $\gamma \in \Gamma$ in $H^1(\Gamma, \mathbb{R})$. Identifying the H-valued representations ρ with $H^1(\Gamma, \mathbb{R})$, the author proves that graph(ρ) acts properly if and only if $\|\rho\| < 1$.