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Review text:

This paper discusses the relationship between two parametrizations of complex projective structures ($\mathbb{C}P^1$-structures) on a closed surface S. Classically, $\mathbb{C}P^1$-structures are parametrized by elements $([M], q)$ of the total space of the cotangent bundle $\pi : T^*\mathfrak{T}(S) \longrightarrow \mathfrak{T}(S)$ of Teichmüller space $\mathfrak{T}(S)$, whereby the point $[M] = \pi(([M], q))$ in Teichmüller space records the *complex structure* underlying the $\mathbb{C}P^1$-structure. The fiber is the vector space $Q(M)$ of holomorphic quadratic differentials $q \in Q(M)$ on the Riemann surface M which may be identified as tangent covectors in $T^*_{[M]}\mathfrak{T}(S)$. The cotangent vector q arises as the Schwarzian differential of the developing map (regarded as an $\pi_1(M)$-equivariant meromorphic function defined on the universsal covering $\tilde{M} \longrightarrow M$).

The second parametrization is due to Thurston and is much more recent. For this a $\mathbb{C}P^1$-structure is realized as a *pleated section* of a flat $\mathbf{H}^3$-bundle over S, in other words, as a pleated map from the universal covering surface $\tilde{S}$ into $\mathbf{H}^3$ which is equivariant with respect to a homomorphism $\phi : \pi_1(M) \longrightarrow \mathrm{PSL}(2, \mathbb{C})$. A pleated map is a piecewise totally geodesic embedding, which is bent along a geodesic lamination. The intrinsic geometry on the pleated surface is hyperbolic geometry, and is recorded by a (probably different) point $[M']$ in $\mathfrak{T}(S)$. (Here $\mathfrak{T}(S)$ is regarded as the space of *hyperbolic* structures rather than complex structures.) The extrinsic geometry is recorded by an invariant transverse measure to this lamination. Therefore $\mathbb{C}P^1$-structures are parametrized by the Cartesian product $\mathfrak{T}(S) \times \mathcal{ML}(S)$ where $\mathcal{ML}(S)$ is Thurston's space of measured geodesic laminations.

Monodromy defines a map

$$\mathrm{mon} : \mathbb{C}P^1(S) \longrightarrow \mathrm{Hom}(\pi_1(S), \mathrm{PSL}(2,\mathbb{C}))/\,\mathrm{PSL}(2,\mathbb{C})$$

from the moduli space $\mathbb{C}P^1(S)$ of $\mathbb{C}P^1$-structures on S. In general this map is not proper. The author geometrically proves Kapovich's result (*On monodromy of complex projective structures*, Inv. Math. **119** (1995), 243–265) that, for a fixed Riemann surface M, the restriction $\mathrm{mon}\,|_{Q(M)}$ is proper (although mon is *not* proper. More generally she proves that for any compact $K \subset \mathfrak{T}(S)$, the restriction $\mathrm{mon}\,|_{\pi^{-1}(K)}$ is proper.

The monodromy map is more directly related to (and more eaily computable in) Thurston's parametrization using bending than the classical parametrization in terms of quadratic differentials. The proofs use the author's previous work (*Grafting, harmonic maps and projective structures on surfaces*, J. Diff. Geo. **47** (1997), 339–319) to relate extremal length in the hyperbolic structure underlying the bent structure to the length of laminations. An interesting new concept is the extension of the length function associated to a measured geodesic lamination μ to a function ℓ_μ on the subset of $\mathrm{Hom}(\pi_1(S), \mathrm{PSL}(2,\mathbb{C}))/\,\mathrm{PSL}(2,\mathbb{C})$ consisting of nonelmentary representations, which is proved to be locally bounded.