Abstract

On Weakly Wandering Vectors And Weakly Independent Partitions

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(Joint work with A. del Junco)

In the early 70-ies U.Krengel studied the phenomenon of wandering of vectors under the actions of powers of a unitary operator in Hilbert space, and of independence of images of a partition under powers of an invertible measure-preserving transformation.

A non-zero vector $f \in H$ is called weakly wandering with respect to U if there exists an increasing sequence $\{n_i\}$ such that the vectors $U^{n_i} f$ are pairwise orthogonal. Krengel proved that if U is weakly mixing then the set of weakly wandering vectors is dense in H. He also proved a parallel result for partitions. A partition P of a probability space X is called weakly independent for a m.p.t. $T : X \to X$ if all the partitions $T^{n_i}P$ are independent for some sequence $\{n_i\}$. It was shown by Krengel that weakly independent partitions are dense in the set of all partitions.

Later on, numerous refinements and generalizations of this result were obtained by V. Bergelson – I. Kornfeld – A. Leibman – B. Mityagin, A. del Junco, and others. In particular, it was shown that wandering/independence can be forced along IP-sets (sets of all possible finite sums of the elements of a sequence), and that the phenomenon holds for certain group actions as wells.

I will present the wandering result for any countable Abelian group, and talk on the progress on the generalization of the result about partitions, for free actions of amenable groups, not necessarily Abelian.

Although the two questions (of wandering vectors and of independent partitions) are close in spirit, the proofs are obtained by completely different methods. In the proof for partitions the following lemma about extentions of measures, or, in other words, about building a stationary process from its marginals, plays an important role.

Suppose G is a countable amenable group, Ω is a finite set. Let m be any shift-invariant measure on Ω^P (where $P \subset G$ is finite). Then m can be extended to a shift-invariant measure on Ω^G, provided m is close enough to the product measure. The required degree of proximity depends only on the one-dimensional marginal of m, and on the cardinality of P, but not on its structure. (Here we call a measure *product measure* if it is the product of its one-dimensional marginals. We say that a measure on Ω^P is (left) *shift-invariant* if its marginals on Ω^Q and Ω^{gQ} are obtained from one another by shift for any $Q, gQ \subset P$.)