Computation of Entropy in Statistical Mechanics and Information Theory

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Abstract

The exponential growth rate h (with respect to the natural logarithm) of the number of configurations on $\mathbb{Z}^d$ with respect to a given subshift of finite type arises in the theory of various physical phenomena. In physics e^h is viewed as the entropy (per atom) of the corresponding hard model, in mathematics h is called the topological entropy, and in information theory h (with respect to $\log_2$) is called the multi-dimensional capacity. In the one-dimensional case ($d = 1$) e^h is the spectral radius $\rho(A)$ of the corresponding transfer matrix. There are very few 2-dimensional models where the value of h is known in closed form. In all other cases there are estimates based on (a) asymptotic expansions; (b) Monte-Carlo methods; (c) bounds.

In this talk we give the complete up-to-date theory for computing h by lower and upper bounds. It refines the techniques described in [Friedland, Proc. MTNS 2002] by using an automorphism subgroup of a certain graph. As a demonstration of these techniques, we compute the topological entropy of the monomer-dimer system in a plane grid with 8-digit precision. We compare our results to the known results.

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