

# CRITICAL BEHAVIOR OF GAUSSIAN RANDOM MATRICES WITH EXTERNAL SOURCE

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ABSTRACT. We consider the Gaussian ensemble of Hermitian random matrices with external source,

$$\frac{1}{Z_N} e^{-N\text{Tr}(\frac{1}{2}M^2 - AM)} dM$$

where  $A$  has two distinct eigenvalues  $\pm a$  of equal multiplicity. This model exhibits a phase transition at  $a = 1$ , since the eigenvalues of  $M$  accumulate on two intervals for  $a > 1$ , and on one interval for  $0 < a < 1$ . The main goal of the present work is to develop the Riemann-Hilbert approach to the double scaling limit of the eigenvalue correlation functions in the vicinity of the critical point  $a = 1$ . Earlier the double scaling limit has been studied in the works of Brezin and Hikami and of Tracy and Widom, who used the contour integral formula for the reproducing kernel. Our approach is based on a  $3 \times 3$  Riemann-Hilbert problem and the Deift/Zhou steepest descent method. The advantage of this approach is that it allows an extension to non-Gaussian ensembles of Hermitian random matrices. We prove that the limiting eigenvalue correlations are expressed in terms of the sine kernel in the bulk of the spectrum, in terms of the Airy kernel at the edge, and in terms of the Pearcey kernel in the double scaling limit.

This is a joint work with Arno Kuijlaars.

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