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Erdos measures and Markov chains.

Abstract . We consider random variable $\zeta = \sum_{i=1}^{\infty} \omega_i \rho^i$, where the process $\omega_1, \omega_2, ...$ is a stationary ergodic aperiodic 2-step Markov chain, taking values in $\{0,\ 1\}$ and $\rho = 1/\text{golden ratio}$. The distribution of ζ must have no discrete spectrum and is either purely singular continuous or purely absolutely continuous. We give the description of conditions under which ζ is absolutely continuous.

Fibonacci expansion for random point $\rho\zeta$ from $[0,1]$ is of form $\eta_1\rho + \eta_2\rho^2 + ...$, where random variables η_k are taking values in $\{0,\ 1\}$ and $\eta_k\eta_{k+1} = 0$. Infinite random word $\eta = \eta_1\eta_2...\eta_n...$ takes values in Fibonacci compactum and defines an Erdos measure $\mu(A) = P(\eta \in A)$. Invariant Erdos measure is the shift-invariant measure with respect to which Erdos measure is absolutely continuous.

We show that the invariant Erdos measure is an one-block factor of a regular Markov chain.This gives an ergodic properties of the invariant Erdos measure. For some cases we calculate the Hausdorff dimension of the Erdos measure.

If $\omega_1, \omega_2, ...$ are independent random variables, then the number of states of the corresponding regular Markov chain equals 5. If $\omega_1, \omega_2, ...$ is a Markov chain then the number of states of the corresponding regular Markov chain equals 7.

The talk is based on joint works with Zina Bezhaeva.