QUANTIZATION DIMENSION AND ERGODIC MAKOV MEASURE ASSOCIATED WITH RECURRENT SELF-SIMILAR SET

MRINAL KANTI ROYCHOWDHURY
DEPT OF MATHEMATICS
THE UNIVERSITY OF TEXAS-PAN AMERICAN

Abstract: Given a Borel probability measure μ on $\mathbb{R}^d$, a number $r \in (0, +\infty)$ and a natural number $n \in \mathbb{N}$, the nth *quantization error* of order r for μ, is defined by

$$e_{n,r} = \inf\{\big(\int d(x, \alpha)^r d\mu(x)\big)^{\frac{1}{r}} : \alpha \subset \mathbb{R}^d, \text{ card}(\alpha) \leq n\},$$

where $d(x, \alpha)$ denotes the distance from the point x to the set α with respect to a given norm $\| \cdot \|$ on $\mathbb{R}^d$. Note that if $\int \|x\|^r d\mu(x) < \infty$ then there is some set α for which the infimum is achieved. The upper and lower quantization dimensions for μ of order r are defined by

$$\overline{D}_r(\mu) := \limsup_{n\to\infty} \frac{\log n}{-\log e_{n,r}}; \quad \underline{D}_r(\mu) = \liminf_{n\to\infty} \frac{\log n}{-\log e_{n,r}}.$$

If $\overline{D}_r(\mu)$ and $\underline{D}_r(\mu)$ coincide, we call the common value the quantization dimension of μ of order r and is denoted by $D_r(\mu)$. One sees that the quantization dimension is actually a function $r \mapsto D_r$ which measures the asymptotic rate at which $e_{n,r}$ goes to zero. If D_r exists, then asymptotically

$$\log e_{n,r} \sim \log(\frac{1}{n})^{1/D_r}.$$

Let $P = [p_{ij}]_{1\leq i,j\leq N}$ be an $N \times N$ irreducible row stochastic matrix and $X \subset \mathbb{R}^d$ be a compact set such that $X = \text{cl}(\text{int}X)$. To each p_{ij} if $p_{ij} > 0$, let us associate a contractive similitude S_{ij} mapping X into X with the similarity ratio s_{ij} $(0 < s_{ij} < 1)$. Then the collection $\{X, S_{ij}, p_{ij} : p_{ij} > 0, 1 \leq i, j \leq N\}$ is called a *recurrent iterated function system* (RIFS) of similarity mappings. Let us now consider the ergodic Markov measure ν on the coding space, and take its image measure $\mu := \nu \circ \pi^{-1}$ on the recurrent self-similar set via the coding map π. In this paper, I have determined the quantization dimension function for the probability measure μ, and established a relationship between the quantization dimension function and the temperature function of the thermodynamic formalism that arises in multifractal analysis.