TOPOLOGY QUALIFYING EXAM

AUGUST, 2001

(1) **(Math 730)**Compute the fundamental group of the open subset Ω of $\mathbb{R}^3$ obtained by removing the three coordinate axes.

(2) **(Math 730)**We suppose that all topological groups are semi-locally path-connected so that the theory of covering spaces applies.
   (a) Show that a discrete normal subgroup N of a connected topological group G lies in the center of G.
   (b) Let G, H be connected n-dimensional topological groups and $f : G \longrightarrow H$ a homomorphism which is a covering space. Show that the kernel of f is abelian.
   (c) Show that the fundamental group of a connected topological group must be abelian.

(3) **(Math 730)**
   (a) Let X, Y be topological spaces. Suppose that X is compact and Y is Hausdorff. Let $f : X \longrightarrow Y$ be continuous and bijective. Prove that f is a homeomorphism.
   (b) Find a counterexample when X is only assumed to be locally compact.

(4) **(Math 734)**

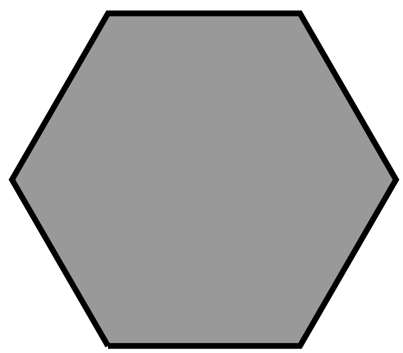
Let $f : X \to \mathbb{CP}^n$ be a continuous map from a CW complex X to complex projective n-space. Suppose that the map $f_* : H_{2n}(X; \mathbb{Z}) \to H_{2n}(\mathbb{CP}^n; \mathbb{Z})$ is nonzero.

(a) If β is the generator of $H^{2n}(\mathbb{CP}^n; \mathbb{Z})$, show that $f^*(\beta) \neq 0$.

(b) Show that $H^2(X; \mathbb{Z})$ and $H_2(X; \mathbb{Z})$ are both nontrivial groups. (Hint: for the last part tensor with $\mathbb{Q}$ to obtain that $f^*\beta$ is not torsion.)

(5) **(Math 734)**

Let X be the topological space obtained by taking a solid regular hexagon and identifying the opposite edges by parallel translation.



Calculate the integral homology of X.

(6) **(Math 734)**

Recall that the connected sum $M\#N$ of two oriented n-manifolds is obtained by removing an open n-disk from each and identifying the boundaries of the disks by an orientation-preserving homeomorphism.

(a) Express the Euler characteristic $\chi(M\#N)$ in terms of $\chi(M)$ and $\chi(N)$.

(b) Suppose that $n > 2$. Express the fundamental group $\pi_1(M\#N)$ in terms of $\pi_1(M)$ and $\pi_1(N)$.