MATH 734 Ph.D. QUALIFYING EXAM

January, 1990

1. Let $a : S^2 \to S^2$ be the antipodal map and define

$$X = S^2 \times S^2/(x, y) \sim (a(x), a(y)).$$

   a) Compute the fundamental group of X and the Euler characteristic of X.

   b) Show that X is orientable.

   c) Compute the integral cohomology ring of X.

2. Let $f_n : S^1 \to S^1$ be the n-fold covering map defined by $f_n(e^{i\theta}) = e^{in\theta}, 0 \leq \theta < 2\pi$, $n \in \mathbb{Z}$. Let M_n be the cell complex obtained by attaching the closed 2-disk D^2 to S^1 with attaching map f_n.

   a) Compute the integral homology of M_n.

   b) Show that for homological reasons M_n is not a manifold for $|n| \neq 2$.

3. Let G be a topological group manifold, i.e., G is a topological group which is a manifold in the same topology. Suppose G is path connected.

   a) Show that G is orientable.

   b) Let $\mu : G \times G \to G$ be the map given by group multiplication, and for $g \in G$ define $\mu_g : G \to G$ by $\mu_g(x) = \mu(g, x)$. Show that μ_g is homotopic to the identity map.

   c) Use the Lefschetz fixed point theorem to show that $\chi(G) = 0$.