MATH 734 Ph.D. QUALIFYING EXAM

January, 1992

1. (a) Let X be an orientable closed manifold of dimension n. Suppose $\tau : X \to X$ is a continuous map and τ^k is the identity map for some odd integer k. Show that $H^n(\tau)$ is the identity map.

   (b) Let Σ be the closed orientable surface of genus two. Show that if $\tau : \Sigma \to \Sigma$ has order 3 then τ has a fixed point.

2. For $x \in \mathbb{R}$, let $[x]$ be the greatest integer less than or equal to x. Let $\Delta_1 = \{(t_0, t_1) \in \mathbb{R}^2 \mid t_0 + t_1 = 1,\ t_i \geq 0\}$ be the standard 1-simplex; let $e_0 = (0,1)$ and $e_1 = (1,0)$. Let $\sigma : \Delta_1 \to \mathbb{R}$ be a singular 1-simplex.

   (a) Show that the 1-cochain $f \in C^1(\mathbb{R}, \mathbb{Z})$ defined by
   $$f(\sigma) = [\sigma(e_1) - \sigma(e_0)]$$
   is not a cocycle.

   (b) Show that the 1-cochain $g \in C^1(\mathbb{R}, \mathbb{Z})$ defined by
   $$g(\sigma) = [\sigma(e_1)] - [\sigma(e_0)]$$
   is a cocycle.

   (c) Determine whether g is a coboundary.

3. (a) Show (by giving an explicit decomposition) that $S^2 \times T^2$ is a CW complex (or cellular complex).

   (b) Compute the cohomology ring $H^*(S^2 \times T^2, \mathbb{Z})$.