MATHEMATICS 734: ALGEBRAIC TOPOLOGY
DR. ROSENBERG
MID-TERM EXAMINATION
MONDAY, MARCH 18, 1991

Instructions

Show all your work in your exam booklet. The more intermediate steps you show in a problem, the greater the likelihood of your receiving partial credit if the final answer is not completely correct. Unless otherwise stated, you may appeal to any of the theorems proved in class or in the textbook. The point values of the problems are indicated.

1. (**Short answer**, 7 points each) Give brief definitions for each of the following:
   (a) chain map (or, equivalently, morphism of chain complexes);
   (b) chain homotopy;
   (c) chain homotopy equivalence;
   (d) relative (singular) cycle for a pair (X, A), $A \subseteq X$;
   (e) Mayer-Vietoris sequence.

2. (30 points) Suppose X is a topological space, R is a principal ideal domain (this implies every submodule of a free R-module is free), and X is homotopy-equivalent to an n-dimensional CW-complex. Show that $H_j(X; R)$ vanishes for $j > n$ and is a free R-module for $j = n$.

3. (35 points) Show using homology that there is no retraction from $\mathbb{RP}^4$ to $\mathbb{RP}^3$.