POTENTIALS AND FLUX ON MARKOV RECURRENT CHAINS

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ABSTRACT. Let (P,T) be a Markov chain where T is a countable set of states and $P = \{p(v,u)\}_{v,u \in T}$ is the matrix of transition probabilities on the states. We shall consider only the case that this chain is a tree; i.e. given any states u, v there exists a unique sequence of states $u = u_0, u_1, \ldots u_n = v$ such that $p(u_{k-1}, u_k) > 0$ for $k = 1, \cdots, n$. Thinking of P as a random walk on T, we may consider the notions of transience and recurrence. Potential theory is usually done on transient chains, but in this work we find a replacement notion for potential – called an H-potential and investigate its properties. We also compare this to earlier notions of potentials on recurrent Markov chains due to Kemeny, Snell, Kesten, Orey, and Spitzer, among others.

We also define $H-{\rm Green}$ functions and c consider a noton of flux for super-harmonic functions.