MATH 416, Spring 10, Midterm 2 Review

1. Write explicitely the matrix of a 4×4 DFT. Apply it to a vector $(0, 1, 0, -1)$.

2. Find the Lagrange polynomial through points $(1, 2)$, $(2, 5)$, $(3, 4)$.

3. Suppose that $f(x) = mx$ for some constant m. Show that for any sampling of f, the piecewise linear approximation exactly equals f.

4. Show that the set of functions $\{\sqrt{2}\sin(\pi n t) : n = 1, 2, 3, \ldots\}$ is orthonormal with respect to the real inner product which is defined as: $\langle f, g\rangle = \sum_{k=0}^{N-1} f(k)g(k)$.

5. Show that the set of vectors $\omega_n \in \mathbb{C}^N, n = 0, \ldots, N-1$, where $\omega_n(k) = 1/\sqrt{N}e^{2\pi i n k/N}$, is an orthonormal basis for $\mathbb{C}^N$ with respect to the complex inner product which is defined as: $\langle f, g\rangle = \sum_{k=0}^{N-1} f(k)\overline{g(k)}$.

6. Write explicitely the matrix of a 5×5 DCT III. Apply it to a vector $(1, 0, -1, 0, 1)$.

7. What is the matrix of the square of $N \times N$ DFT?

8. What is the matrix of the $N \times N$ inverse DCT-IV transform?

9. Find the expansion in Chebyshev polynomials $T_0(x), T_1(x), T_2(x)$ of the function $f(x) = 1 + x^2$ dened for $x \in [-1, 1]$.

10. Prove that the $N \times N$ discrete Hartley transform matrix is symmetric and unitary.