Solutions for HW1

**Problem 1.** Suppose that the collection of vectors $\{f_1, ..., f_n\}$ is a strong finite frame. Then there exist constants $A < B$ which obviously satisfy the inequality $A \leq B$ so every strong finite frame is a finite frame;
now suppose that $\{f_1, ..., f_n\}$ is a finite frame. Then we have constants $A \leq B$. To obtain constants $A_1$, $B_1$ s.t. $A < B$ we may either decrease $A$ or increase $B$ (take $A_1 = A < B+1 = B_1$ for example). Thus it is possible to find constants satisfying a strong finite frame condition.

**Problem 2.** Here is a sample code written in Matlab: N=32; (see the attachement).

**Problem 3.** FFT algorithm code was based on the pseudocode given on the page 95 of "Mathematics for Multimedia" .

**Problem 4.** To show that given vectors form the orthonormal we prove that $< \omega_m, \omega_n >$ is 1 for $m = n$ (the norm) and 0 otherwise:

$$< \omega_n, \omega_n >= \sum_{k=1}^{N} \frac{1}{\sqrt{N}}(cos(2\pi nk/N) + i\sin(2\pi nk/N))\frac{1}{\sqrt{N}}(cos(2\pi nk/N) - i\sin(2\pi nk/N)) =$$

$$= \sum_{k=1}^{N} \frac{1}{N}(cos^2(2\pi nk/N) - i^2\sin^2((2\pi nk/N)) = \frac{1}{N}\sum_{k=1}^{N} 1 = N/N = 1$$

$$< \omega_m, \omega_n >= \sum_{k=1}^{N} \frac{1}{\sqrt{N}}\frac{1}{\sqrt{N}}e^{2\pi i(n-m)k} = \text{ (as the sum of the N-th roots of unity) } = 0$$

Which concludes the proof.