MATH 464, FALL 2010, MIDTERM 1 SAMPLE PROBLEMS

1) Express the Inverse Fourier Transform of the following functions, in terms of the Inverse Fourier Transform of f:

a) $kf(kx)$;

b) $f'(x)$.

2) Express the Fourier Transform of the following function $ae^{2\pi iabx}f(ax-c)$, in terms of the Fourier Transform of f. (Here a, b, c are positive constants.)

3) Find explicitly the Fourier Transform of the function $0.5e^{\pi ix}\chi_{[4,6]}(x)$ and simplify it, if possible.

4) Calculate explicitly the following convolution: $h(x) = \chi_{[0,1]} \star \chi_{[0,1]}(x)$. Find the Fourier Transform of h.

5) Let f be a purely imaginary function. Verify whether or not $\hat{f}$ is odd, even, or neither.

6) Calculate the following integral

$$\int_{-\infty}^{\infty} \frac{\sin^2(2\pi Tx)}{\pi^2 x^2} dx.$$

7) Calculate the Fourier coefficients of the following $2T$-periodic, locally integrable functions:

a) $f(x) = \begin{cases} 1, & x \in [3T/2, 5T/2), \\ -1, & x \in [5T/2, 7T/2); \end{cases}$

b) $h(x) = \begin{cases} 1, & x \in [-T/2, T/2), \\ 0, & x \in [-T, -T/2) \cup [T/2, T) \end{cases}.$

8) Prove that the operation of taking the $2T$-periodic Fourier Transform of a square-summable sequence $f = \{f[n] : n \in \mathbb{Z}\} \subset \ell^2(\mathbb{Z})$, i.e., a mapping

$$f \mapsto \left\{ \hat{f}(\xi) = \sum_{n \in \mathbb{Z}} f[n] e^{-\pi i n \xi / T} : \xi \in [-T, T) \right\},$$

is a linear transformation.

9) Find the sum of the series:

$$1 + \sum_{n \in \mathbb{Z} \setminus \{0\}} \frac{\sin^2(\pi n/2)}{n^2}.$$

10) Define the collection of vectors:

$$v_n(m) = \sqrt{2/N}\, \sin(\pi n m / N),$$

where $m, n = 1, \ldots, N-1$. Prove that these vectors form an orthonormal basis for $\mathbb{R}^{N-1}$.