de L'Hôpital's Rule

Let $\lim_{x\to *}$ denote one of the following:

(1) a classical limit - $\lim_{x\to a}$, where a is a number,

(2) a one sided limit - $\lim_{x\to a^+}$ or $\lim_{x\to a^-}$,

(3) or a limit at infity - $\lim_{x\to +\infty}$ or $\lim_{x\to -\infty}$,

respectively.

For each of these cases let I denote:

(1) union of two open intervals adjacent to a (i.e., the interval (b, c) take away a, where $b < a < c$),

(2) open interval with a as one of it endpoints (i.e., either $I = (a, b)$, or $I = (b, a)$),

(3) or an open half line (i.e., $I = (b, +\infty)$, or $I = (-\infty, b)$),

respectively.

Assume f and g are differentiable on I, and that $g' \neq 0$ on I.

Assume $\lim_{x\to *} f(x) = \lim_{x\to *} g(x) = 0$, or, alternatively, that $\lim_{x\to *} f(x) = \lim_{x\to *} g(x) = \pm\infty$.

Then,

$$\lim_{x\to *} \frac{f(x)}{g(x)} = \lim_{x\to *} \frac{f'(x)}{g'(x)},$$

provided the limit on the right hand side (RHS) exists (this includes, in particular, the case of the RHS limit being equal to $+\infty$ or $-\infty$).