Math 141, FALL 200, MIDTERM 1 Answer Key for Question #4

4. Evaluate the integral $\int \frac{x}{\sqrt{1-x^4}}\,dx$.

Rewrite $\int \frac{x}{\sqrt{1-\left(x^2\right)^2}}\,dx$. Let $u = x^2$, $du = 2x\,dx$, $\frac{1}{2}du = x\,dx$.

Substitute $\int \frac{x}{\sqrt{1-\left(x^2\right)^2}}\,dx = \frac{1}{2}\int \frac{1}{\sqrt{1-u^2}}\,dx$.

Recognize $\int \frac{1}{\sqrt{a^2-u^2}}\,dx = \sin^{-1}\left(\frac{u}{a}\right)+C$ where $a = 1$ and $u = x^2$.

Evaluate $\int \frac{x}{\sqrt{1-\left(x^2\right)^2}}\,dx = \frac{1}{2}\int \frac{1}{\sqrt{1-u^2}}\,dx = \frac{1}{2}\sin^{-1}\left(x^2\right)+C$.

GRADING:
I drew a line down the back of the answer page if it was not used.

no work shown:	–5
incorrect inverse trig:	–5
$\frac{1}{2}\sin^{-1}\left(\underline{\underline{x^4}}\right)+C$:	–4
$\frac{1}{2}\sin^{-1}\underline{\underline{x}}+C$:	–4
severe algebra mistake:	–3
missing coefficient:	–2
missing + C:	–2
not replacing u with x^2:	–2
x and u in same integral	–2
invalid statement off to the side	
e.g. $\frac{1}{1-x^2} = \sin^{-1}$	–1
redundant du and/or dx:	–1
missing dx and/or du:	–1 each
$\frac{1}{2}\sin^{-1}\left(\underline{\underline{\frac{x^2}{1}}}\right)+C$:	–1
didn't do subst. but got $\sin^{-1}x^2$	
all else correct	–10
$\int \frac{1}{\underline{\underline{\sqrt{\frac{1}{x^2}-x^2}}}}\,dx = \sin^{-1}\left(\underline{\underline{x^2}}\right)+C$:	–15
recognizing inverse sin,	
all else wrong	–17
recognizing inverse trig,	
all else wrong	–18