MATH 141, FALL 2008, MIDTERM 3

2) Find the sum of the series $\sum_{n=1}^{\infty}\left(\frac{1}{n+1}-\frac{1}{n+3}\right)$.

4 points

Construct the *j*th partial sum, noting that this is a telescoping series: subsequent fractions zero out.

$$s_j = \left(\frac{1}{2}-\frac{1}{4}\right)+\left(\frac{1}{3}-\frac{1}{5}\right)+\left(\frac{1}{4}-\frac{1}{6}\right)+\left(\frac{1}{5}-\frac{1}{7}\right)+\left(\frac{1}{6}-\frac{1}{8}\right)+\left(\frac{1}{7}-\frac{1}{9}\right)+\ \cdots$$

4 points

$$+\left(\frac{1}{j-2}-\frac{1}{j}\right)+\left(\frac{1}{j-1}-\frac{1}{j+1}\right)+\left(\frac{1}{j}-\frac{1}{j+2}\right)+\left(\frac{1}{j+1}-\frac{1}{j+3}\right)$$

3 points

$$s_j = \frac{1}{2}+\frac{1}{3}-\frac{1}{j+2}-\frac{1}{j+3}$$

Since $\lim_{j\to\infty}\left(\frac{1}{2}+\frac{1}{3}-\frac{1}{j+2}-\frac{1}{j+3}\right) = \frac{3}{6}+\frac{2}{6} = \frac{5}{6}$, $\sum_{n=1}^{\infty}\left(\frac{1}{n+1}-\frac{1}{n+3}\right)=\frac{5}{6}$.

4 points

3 points

2 points