Math 141, Fall 2011, Midterm 1

Problem 1)

(12 points)

The length L of a curve given parametrically by $(x(t), y(t))$ for $a \leq t \leq b$ is $\int_a^b \sqrt{(x'(t))^2 + (y'(t))^2} dt$.

Here, $x(t) = e^t sin(t)$ and $y(t) = e^t cos(t)$ for $0 \leq t \leq \pi$.

So we have $x'(t) = e^t cos(t) + e^t sin(t) = e^t(cos(t) + sin(t))$and $y'(t) = e^t cos(t) - e^t sin(t) = e^t(cos(t) - sin(t))$ for $0 \leq t \leq \pi$.

So $L = \int_0^\pi \sqrt{(e^t(cos(t) + sin(t)))^2 + (e^t(cos(t) - sin(t)))^2} dt$

(8 points)

$= \int_0^\pi e^t \sqrt{cos^2(t) + sin^2(t) + 2cos(t)sin(t) + cos^2(t) + sin^2(t) - 2cos(t)sin(t)} dt$

$= \int_0^\pi e^t \sqrt{2(cos^2(t) + sin^2(t))} dt$, and since $cos^2(t) + sin^2(t) = 1$, we have that

$L = \int_0^\pi \sqrt{2} e^t = \sqrt{2}(e^\pi - e^0) = \sqrt{2}(e^\pi - 1)$.

(5 points)

To find the starting and ending points of this curve, we simply plug in the first and last time values for t in $(x(t), y(t))$ to get that the starting point is $(x(0), y(0)) = (0, 1)$ and that the ending point is $(x(\pi), y(\pi)) = (0, e^{-\pi})$.