

## Problem 1

$$y' = \cos(x)y + \cos x$$

Method (i) :  $P(x) = -\cos x$  (2)

$$Q(x) = \cos(x) \quad (2)$$

$$S(x) = \int P(x)dx = -\sin x \quad (4)$$

$$\begin{aligned} y(x) &= e^{-S(x)} \int e^{-S(t)} Q(t) dt \\ &= e^{\sin x} \int e^{-\sin t} \cos t dt = e^{\sin x} [-e^{-\sin x} + c] = ce^{\sin x} - 1 \quad (10) \end{aligned}$$

$$y(0) = 2 \Rightarrow c = 3 \quad (4)$$

$$\Rightarrow y(x) = 3e^{\sin x} - 1. \quad (3)$$

Method (ii)  $\frac{dy}{dx} = \cos x(y+1)$  (4)

$$\Rightarrow \frac{1}{y+1} \frac{dy}{dx} = \cos x \Rightarrow \int \frac{1}{y+1} \frac{dy}{dx} dx = \int \frac{1}{y+1} dy = \int \cos x dx$$

$$\Rightarrow \ln(y+1) = \sin x + c \quad (12)$$

$$y(0) = 2 \Rightarrow c = \ln(3) \quad (5)$$

$$\Rightarrow y = 3e^{\sin x} - 1 \quad (4)$$