PROBLEM 1, MIDTERM 3 - SOLUTION

In order to show that the limit

$$\lim_{n\to\infty} \sqrt[n]{n^3},$$

it suffices to observe that due to continuity of function $f(x) = x^3$, we have:

$$\lim_{n\to\infty} \sqrt[n]{n^3} = \lim_{n\to\infty} n^{3/n} = \left(\lim_{n\to\infty} \sqrt[n]{n}\right)^3.$$

Now, in class we showed that

$$\lim_{n\to\infty} \sqrt[n]{n} = 1,$$

and so

$$\lim_{n\to\infty} \sqrt[n]{n^3} = 1^3 = 1.$$

You could also write

$$\sqrt[n]{n^3} = e^{\frac{1}{n}\ln(n^3)} = e^{\frac{3}{n}\ln(n)}$$

and follow as in the case of $\lim_{n\to\infty} \sqrt[n]{n}$.

The most typical errors include writing

$$\lim_{n\to\infty} \sqrt[n]{n^3} = n^0, \quad or \quad \infty^0 = 1.$$

20 points. NO PARTIAL CREDIT for this problem.