Rubric for Midterm 2, Problem 2

October 18, 2013

PART A. 15 POINTS.

$$\int \frac{1}{x^2-3x+3}dx \quad = \int \frac{1}{(x-\frac{3}{2})^2+\frac{3}{4}}dx \qquad [4 \text{ points}]$$

Make $u = x - \frac{3}{2}$. Then $du = dx$, so

$$\int \frac{1}{(x-\frac{3}{2})^2+\frac{3}{4}}dx \quad = \int \frac{1}{u^2+\frac{3}{4}}du \qquad [5 \text{ points}]$$

Now, there are two ways to proceed.

(i). Recall that

$$\int \frac{1}{u^2+a^2}dx \quad = \tfrac{1}{a}\arctan(\tfrac{u}{a}) + c \qquad [3 \text{ points}]$$

so, with $a = \dfrac{\sqrt{3}}{2}$,

$$\begin{aligned} \int \frac{1}{u^2+\frac{3}{4}}dx \quad &= \frac{1}{\frac{\sqrt{3}}{2}}\arctan\left(\frac{u}{\frac{\sqrt{3}}{2}}\right) + c && [2 \text{ points}] \\ &= \frac{2}{\sqrt{3}}\arctan\left(\frac{2}{\sqrt{3}}(x-\tfrac{3}{2})\right) + c && [1 \text{ point, final answer}] \end{aligned}$$

Alternatively, (ii). Do the substitution $v = \dfrac{2}{\sqrt{3}}u$. The substitution into the integral and simplification is worth [3 points], and the evaluation to $\dfrac{2}{\sqrt{3}}\arctan(v)+c$ is worth [2 points]. Rewriting as a function of x gives the final answer and is worth [1 point].

PART B. 10 POINTS.

We want to evaluate $\displaystyle\int \frac{1}{x\left(1+(\ln(x))^2\right)}dx$.

Make $u = \ln(x)$. Then $du = \dfrac{1}{x}dx$. [5 points]. Then,

$$\begin{aligned}
\int \frac{1}{x\left(1+(\ln(x))^2\right)}dx &= \int \frac{1}{1+u^2}du && \text{[1 points]}\\
&= \arctan(u) + c && \text{[3 points]}\\
&= \arctan\left(\ln(x)\right) + c && \text{[1 point, final answer].}
\end{aligned}$$