MATH 141, Midterm 3, 11/06/2013

Problem 2

$$\int \frac{e^{2x}+4}{(e^x-1)^2}dx \xrightarrow[du/u=dx]{u=e^x} \int \frac{u^2+4}{u(u-1)^2}du \qquad [2+4]$$

$$\frac{u^2+4}{u(u-1)^2}=\frac{A}{u}+\frac{B}{u-1}+\frac{C}{(u-1)^2} \qquad [2]$$

$$\Rightarrow u^2+4=A(u-1)^2+Bu(u-1)+Cu \qquad [1]$$

$u=0 \Rightarrow\ 4=A$

$u=1 \Rightarrow\ 5=C$

$u=2 \Rightarrow\ 8=A+2B+2C=14+2B\ \Rightarrow B=-3 \qquad [2+2+3]$

$$\Rightarrow\ \int \frac{e^{2x}+4}{(e^x-1)^2}dx=4\int\frac{du}{u}-3\int\frac{du}{u-1}+5\int\frac{du}{(u-1)^2}$$

$$=4\ln|u|-3\ln|u-1|-5(u-1)^{-1}+c \qquad [6]$$

$$=4x-3\ln|e^x-1|-5(e^x-1)^{-1}+c \qquad [3]$$

Note :

1. Some students may try to solve for A,B,C by this procedure :

$$u^2+4=(A+B)u^2-(2A+B-C)u+A \qquad [1]$$

coefficient of x^0 : $A=4$ [2]

coefficient of x^1 : $C=2A-B$ [2]

coefficeint of x^2 : $1=A+B=4+B$ [2]

$\Rightarrow\ A=4,\ B=5,\ C=3.$

2. Based on feedback I got from my section, some students tried long division and arrived at :

$$\frac{e^{2x}+4}{(e^x-1)^2}=1+\frac{3+2e^x}{(e^x-1)^2}=1+2\frac{e^x}{(e^x-1)^2}+\frac{3}{(e^x-1)^2} \qquad \text{... [3]}$$

$$\int 1dx=x \qquad \text{.... [1]}$$

$$\int\frac{2e^x}{(e^x-1)^2}dx=-2(e^x-1)^{-1} \qquad \text{.... [2]}$$